MAL 180 Discrete Mathematics

Minor-I (Sep 2009)

Time: 1 Hour

Max. Marks: 25

Attempt Q-1 and any other three questions.

1. (a) Prove that valuations $PC(X) \longrightarrow \{0, 1\}$ always exist and using this, prove that $PC(X)$ is consistent i.e. $\perp$ is not a theorem of $PC(X)$. You can quote soundness, $PC(X)$ is propositional calculus.

   (b) Using the deduction theorem, prove $\vdash p \Longrightarrow \sim\sim$ in $PC(X)$.
   [Hint: Start with $\{p, \sim p\}$ as the set of hypotheses.]

[4+3=7 Marks]

2. (a) Solve the divide and conquer relation

$$a_n - 7\, a_{\frac{n}{3}} = 2n$$

   where $n = 3^k,\ for\ k \geq 1,\ a_1 = 5/2.$

   (b) Solve the inhomogeneous relation

$$a_n - 5a_{n-1} + 6a_{n-2} = n(n-1),\ for\ n \geq 2$$

   given that $a_0 = 1,\ a_1 = 5.$

[3+3=6 Marks]

3. (a) If $f(n) = 2^n,\ g(n) = 3^n$, prove that f is of order g but g is not of order f. Also, if $f(n) = \log_2(n^x),\ g(n) = \log_e(n)$, prove that f is of order g and g is of order f. Prove that the big O-relation is reflexive and transitive.

   (b) If $a,\ b,\ c$ are positive integer constants and $T : \mathbb{N} \longrightarrow \mathbb{R}$ is positive valued monotone increasing obeying

$$T(n) \leq b,\ for\ n = 1,\ T(n) \leq a.T\left(\frac{n}{c}\right) + bn,\ for\ n > 1.$$

   Show that T must be

   i. in $O(n)$ if $a < c$,
   ii. in $O(n \log n)$ if $a = c$, and
   iii. in $O(n^{\log_c a})$ if $a > c$.

[3+3=6 Marks]

4. (a) If H and K are subgroups of G, then show that $H \cap K$ is also a subgroup of G.

   (b) Find all subgroups of $U(17) = \{1, 2, \ldots, 16\}$ which you know is a group under multiplication modulo 17.

[3+3=6 Marks]

5. (a) Define cyclic group. Give one example. Show that every cyclic group is necessary abelian.

   (b) Let a be an element of a group G s.t. $|a| = n$ and h be any positive integer. Show that $|a^h| = n$ *if and only if* $\gcd(h, n) = 1$.