

Department of Mathematics
MAL 110: Mathematics I
Minor I, September 6, 2009

Samy

Maximum Marks: 25

Time: 1 Hr.

1. Let $f(x)$ be a thrice differentiable function. Using Taylor's theorem, prove that

$$f''(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}.$$

[5]

2. Evaluate the integral

$$\int_0^{\infty} 2^{-9x^2} dx.$$

$\frac{1}{6} \sqrt{\frac{\pi}{\ln 2}}$

[5]

3. Let $f(x)$ be a function defined on $[a, b]$ such that

$$|f(u) - f(v)| \leq |u - v|,$$

for all $u, v \in [a, b]$. Prove that $f(x)$ is continuous on $[a, b]$, and hence show that

$$\left| \int_a^b f(x) dx - (b-a) f(c) \right| \leq (b-a)^2,$$

for every $c \in [a, b]$.

[5]

4. Determine the volume of the solid generated by revolving about x -axis, the region in the first quadrant bounded by y -axis, the line $x + y = 3$ and the parabola $x^2 = 4y$.

[5]

$\frac{12\sqrt{3}\pi}{15}$

5. Let

$$f(x, y) = \begin{cases} \frac{(1 - \cos x)(1 - \cos y)}{xy}, & x \neq 0, y \neq 0 \\ 0, & x = 0 \text{ or } y = 0. \end{cases}$$

Evaluate, if they exist, at $(0, 0)$

(i) $\frac{\partial^2 f}{\partial x \partial y}$ and (ii) $\frac{\partial^2 f}{\partial y \partial x}$.

[5]